per vanishing fringe. As the change of glass path of one beam would have to be deducted from $2R$, a somewhat larger value would be anticipated. Testing the complementary fringes (white light) the passage of about 25 fringes completed the phenomenon after which it paled to whiteness. These 25 fringes passed within $\Delta\alpha = 0.75'$, or per fringe about $0.03'$ or $1.8''$ of arc. Of course this is merely an estimate from the small angles of turn involved.

The complementary fringes with sodium light are available indefinitely. I counted about 100 fringes for an angle of $2.7'$, i.e., $1.6''$ per fringe.

Finally using the spectrum fringes of the spectroscope, about 120 fringes were counted within $3'$, i.e., $1.5''$ per fringe. All of these values are larger than the computed value $\lambda/2R$ without correction, but in view of the large number of fringes within exceedingly small angle $\Delta\alpha$, sharp agreement is not to be expected.

[This note is from a Report to the Carnegie Institution of Washington, now in preparation.]

THE DISPLACEMENT INTERFEROMETRY OF LONG DISTANCES

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Communicated, May 22, 1917

1. *Small Angles.*—In my preceding notes I suggested two methods for the measurement of small angles. The first used an auxiliary mirror and apart from corrections the angle $\Delta\alpha$ over which the auxiliary mirror turns is

$$\Delta\alpha = \Delta N \cos i/2R \qquad (1)$$

where ΔN is the displacement of one of the plane mirrors parallel to itself necessary to restore the achromatic fringes to their former position in the field of the telescope, i the angle of incidence (conveniently 45°), $2R$ the normal distance apart of the (parallel) interfering pencils in the fore and aft direction of the incident beam. In the second method the auxiliary mirror is dispensed with and the rotation of a rigid system of paired mirrors is used. The sensitiveness is half the preceding.

2. *Distances.*—Suppose now that the paired mirrors near the telescope confront but a part of the area of the objective and that the telescope can therefore look over the mirrors directly into the region beyond. (A series of small mirrors or reflecting prisms may be employed

to the same purpose; or the mirrors may both be half silvered and transparent.) The telescope now contains two images, the first due to rays (K) entering it directly, the second due to rays (L) reflected into it by the mirrors of the interferometer. Suppose the object seen lies at infinity like a star, that its two images are made to coincide by adjusting the angle α, and that the achromatic fringes have been brought into the field by adjusting the micrometer displacement N.

Now let the angle α be changed by $\Delta\alpha$ until the two images of an object M, at a measurable distance d, coincide. Displace the micrometer mirror by ΔN until the achromatic fringes are restored to their former position. Let b be the effective distance apart of the paired mirrors in the direction right and left to the observer or transverse to the impinging rays (L) and finally s the angle at the apex of the triangle of sight on the base b; i.e. the small angle between the present rays KL. Then

$$d = b \cos g \quad s = b \cos g \quad 2\Delta\alpha = b/2\Delta\alpha \tag{2}$$

(nearly) by the laws of reflection. Hence from equation (1)

$$d = bR/\Delta N \cos i \tag{3}$$

Here $2bR$ is the area of the *ray parallelogram* of the interferometer. Using the constants of my apparatus, let $i = 45°$, $R = 10$ cm., $b = 200$ cm., $\Delta N = 10^{-4}$ cm., the latter being the smallest division on the micrometer. Hence

$$d = 200 \times 10/10^{-4} \times 0.71 = 2.8 \times 10^{7} \text{ cm.} = 280 \text{ kilometers.}$$

or about 170 miles is the limit of measurement of the apparatus.

3. *Performance.*—Again from equation (3) the sensitiveness $\delta(\Delta N)/\delta d$, since

$$\delta d = (d^2 \cos i/bR)\delta(\Delta N) \tag{4}$$

is inversely proportional to the square of the long distance d and the area of the ray parallelogram $2bR$. Thus with the above constants, if d is 2 kilometers, $\delta(\Delta N) = 10^{-4}$ cm., then

$$\delta d = (2 \times 10^5)^2 \times 0.71 \times 10^{-4}/200 \times 10 = 14 \times 10^2 \text{ cm.} = 14 \text{ meters.}$$

Thus an object at about a mile should be located to about 30 feet. Per fringes of mean wave length λ, moreover, $\delta d = \lambda d^2/2bR$, the placement should be about 6 meters at 2 kilometers. I have stated the case, of course, merely for the interferometer, not for subsidiary optical appurtenances.